Maxwell-Chern-Simons Hydrodynamics

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Maxwell-Chern-Simons Electrodynamics

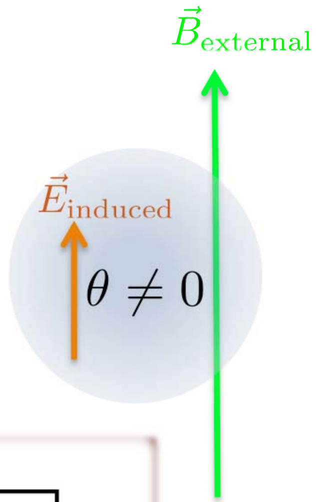
$$\mathcal{L}_{\mathrm{QCD+QED}} = \sum_f \bar{\psi}_f \left[i\gamma^\mu (\partial_\mu - igA_\mu^a t^a - iq_f A_\mu) - m_f \right] \psi_f$$
$$-\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} - \frac{\theta}{32\pi^2} g^2 G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Through quark loops, the electromagnetic sector is effectively

$$\mathcal{L}_{\mathrm{MCS}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu - \frac{c}{4} \theta(x,t) F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

(Talks by B. Mueller & M. Stephanov)

Axial anomalies incorporated here!



$$\boxed{\vec{\nabla} \cdot \vec{E} = \rho - c \vec{\nabla}\theta \cdot \vec{B}}$$

$$\boxed{\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(\dot{\theta} \vec{B} + \vec{\nabla}\theta \times \vec{E} \right)}$$

$$\boxed{\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

Maxwell-Chern-Simons Hydrodynamics

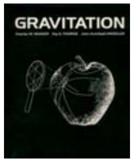
Let's couple quarks to photons in the hydrodynamic picture

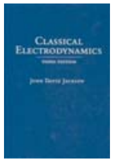
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - g^{\mu\nu} P \qquad \Theta^{\mu\nu} = F^\mu_\lambda F^{\lambda\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

$$(\,\partial_\mu \Theta^{\mu\nu} = -F^{\nu\lambda} J_\lambda - cF^{\nu\lambda}\widetilde{F}_{\lambda\rho}\partial^\rho\theta\,)$$

Energy-momentum conservation

$$\partial_\mu (T^{\mu\nu} + \Theta^{\mu\nu}) = 0$$





Hydrodynamics with Axial Anomalies

$$\boxed{\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\lambda} J_\lambda + cF^{\nu\lambda}\widetilde{F}_{\lambda\rho}\partial^\rho\theta \\ J^\mu &= n u^\mu \end{aligned}}$$

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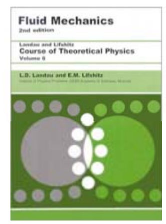
Maxwell-Chern-Simons Hydrodynamics

Using the definitions $E^{\mu} \equiv F^{\mu\nu} u_{\nu}, \; B^{\mu} \equiv \widetilde{F}^{\mu\nu} u_{\nu}$

$$\partial_{\mu} T^{\mu\nu} = nE^{\nu} - cE^{\lambda} B_{\lambda} u^{\nu} u_{\rho} \partial^{\rho} \theta$$



contract with $\Delta^{\alpha}_{\nu} = \delta^{\alpha}_{\nu} - u^{\alpha} u_{\nu}$

contract with u_{ν}

Euler's Equation

$$wDu^{\alpha} - \Delta^{\alpha}_{\nu} \partial^{\nu} P = nE^{\alpha}$$

$\epsilon + P$ $\quad u_{\mu} \partial^{\mu}$

Entropy Equation

$$\partial_{\mu} s^{\mu} = -\frac{1}{T} cE^{\lambda} B_{\lambda} u_{\rho} \partial^{\rho} \theta$$

su^{μ}

$$d\left(\frac{\epsilon + P}{n}\right) = Td\left(\frac{s}{n}\right) + \frac{1}{n} dP$$

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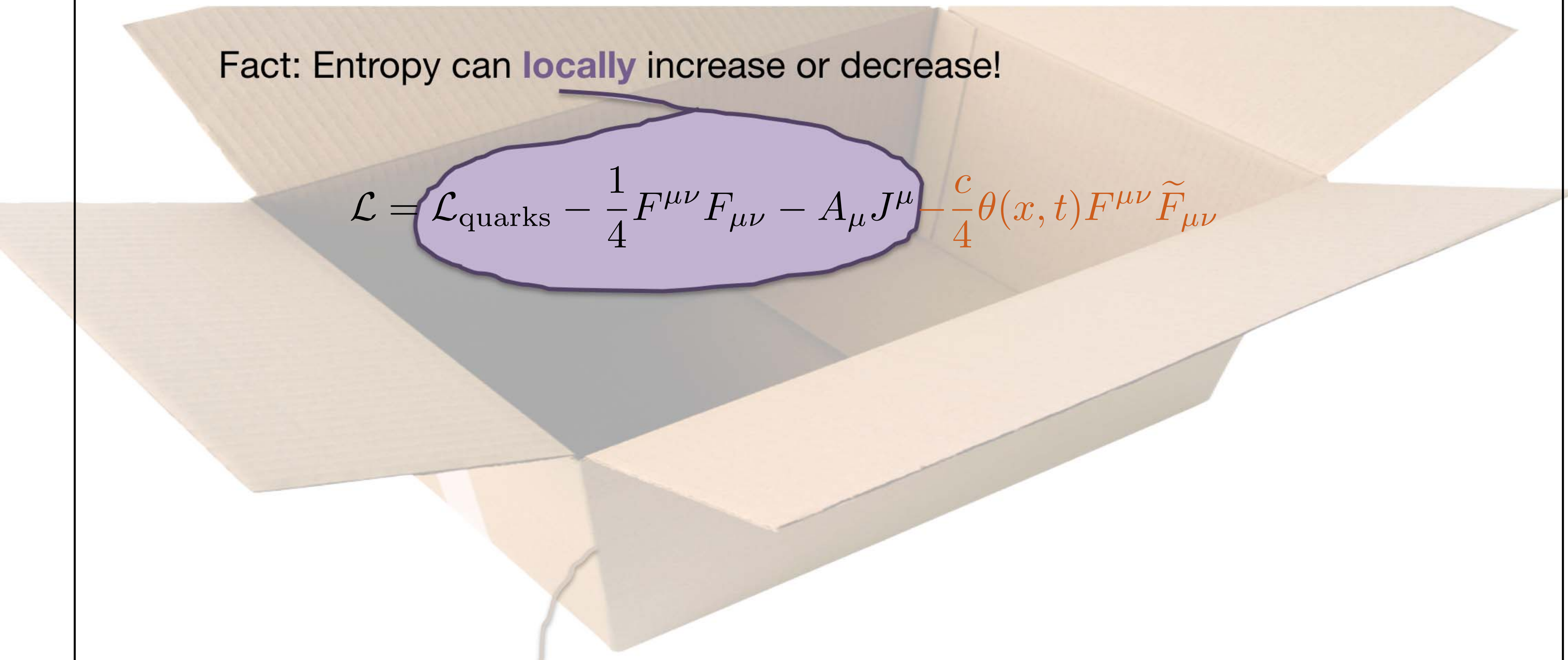
Entropy

Entropy Equation

$$\partial_\mu s^\mu = -\frac{1}{T} c E^\lambda B_\lambda u_\rho \partial^\rho \theta$$



Fact: Entropy can **locally** increase or decrease!

$$\mathcal{L} = \mathcal{L}_{\text{quarks}} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu - \frac{c}{4} \theta(x,t) F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

Though, the total entropy should be constant!

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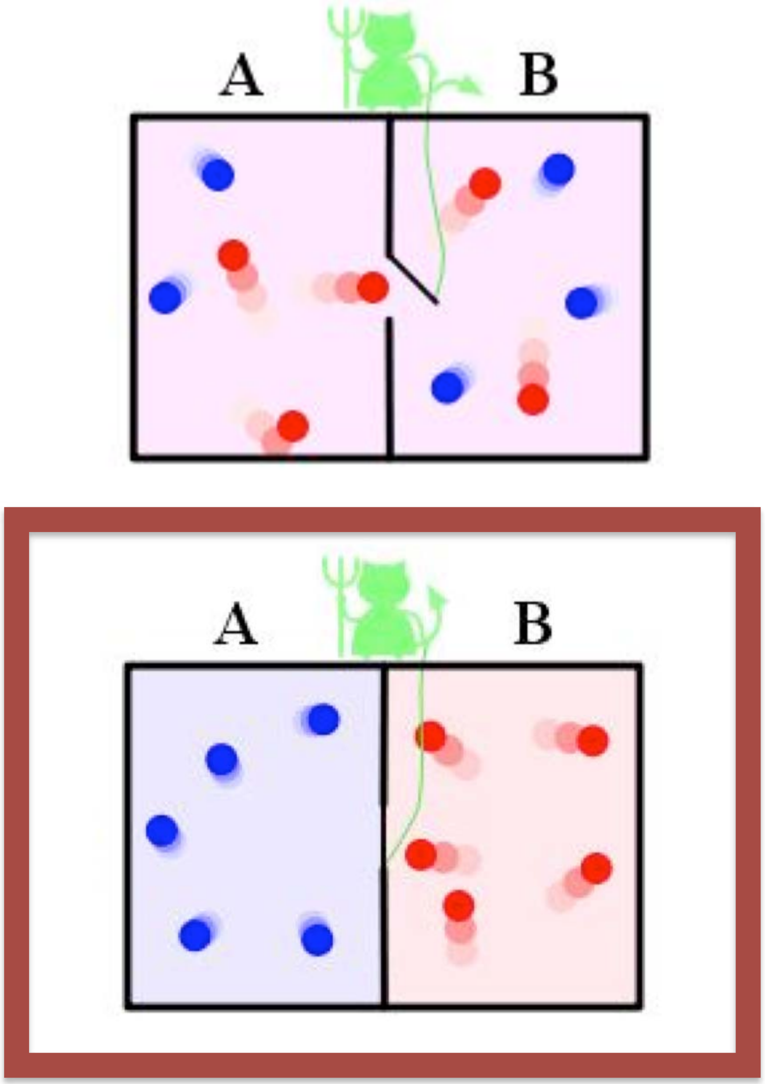
Maxwell's Demon



$\partial_\mu s^\mu \geq 0$

Entropy Revisited

$$\Omega = -P = \frac{T}{V}\ln Z = \frac{T}{V}\ln\left[\exp\left(-\frac{V}{T}c\theta E^{\lambda}B_{\lambda}\right)\int[\mathcal{D}q][\mathcal{D}\bar{q}][\mathcal{D}A_{\mu}]\exp\left(\int_0^{\beta}\mathrm{d}\tau\int\mathrm{d}^3x\mathcal{L}'\right)\right]$$

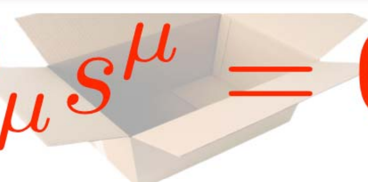
Modified Gibbs relation

$$\left(\frac{\partial\Omega}{\partial\theta}\right)_{T,\mu} = -cE^{\lambda}B_{\lambda}$$

$$d\left(\frac{\epsilon+P}{n}\right) = Td\left(\frac{\sigma}{n}\right) + \frac{1}{n}dP + \frac{1}{n}R_{\theta}d\theta$$

(see Aguiar, Fraga & Kodama)

$$nu^{\mu}\left(\partial_{\mu}\left(\frac{\epsilon+P}{n}\right) - \frac{1}{n}\partial_{\mu}P + \frac{1}{n}cE^{\lambda}B_{\lambda}\partial_{\mu}\theta\right) = 0$$



$$\boxed{\partial_{\mu}s^{\mu} = 0}$$

A Simple Picture

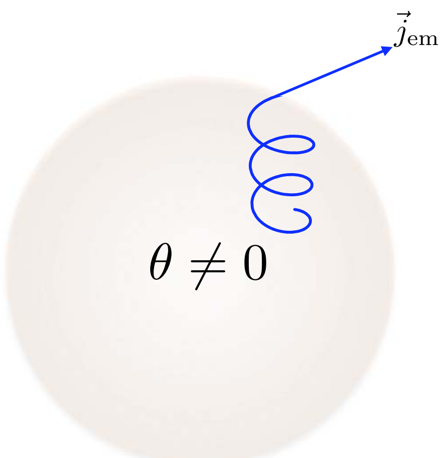
$$D\epsilon + w\partial_\mu u^\mu = -cE^\lambda B_\lambda D\theta$$

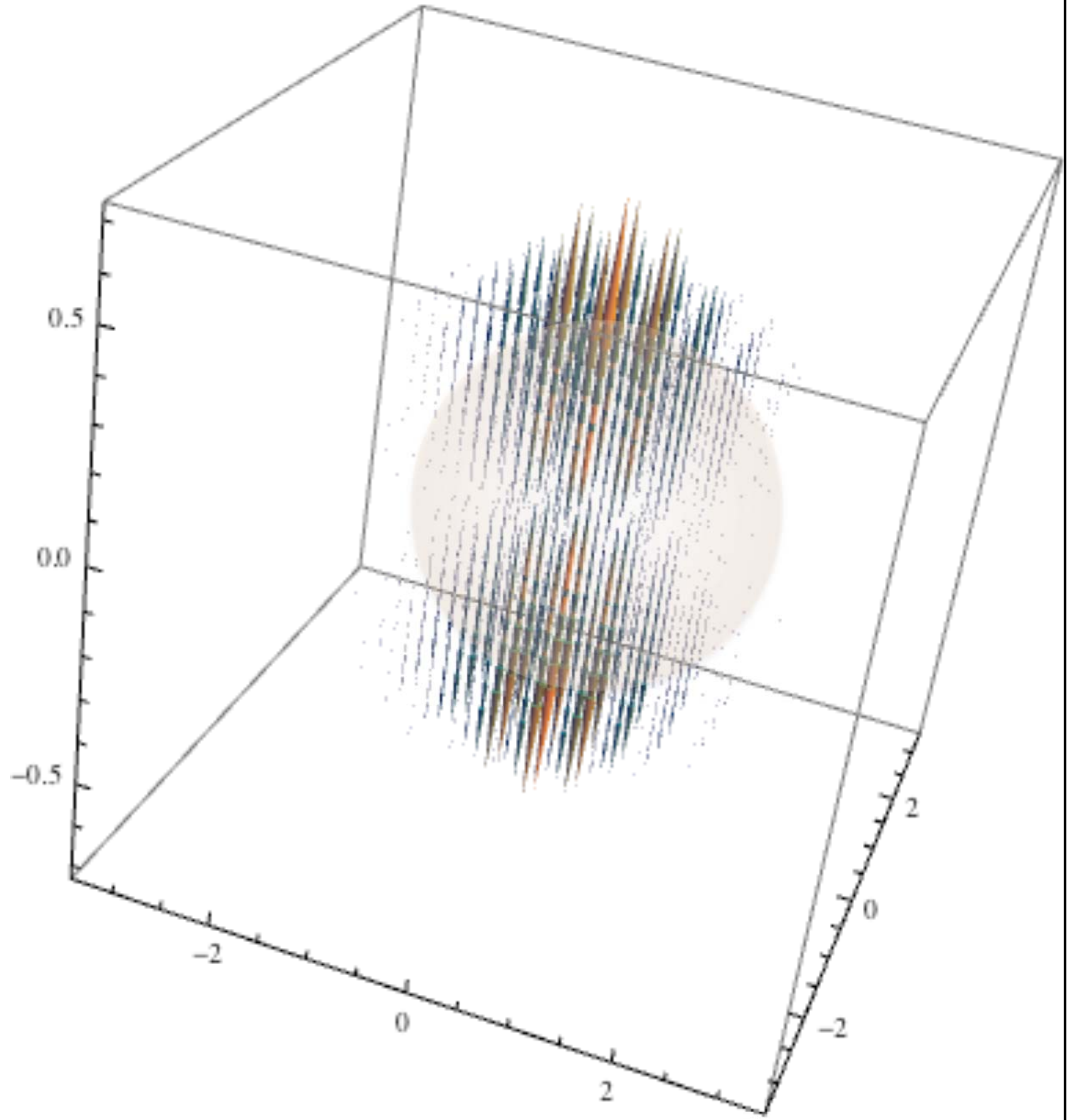
Assumptions:

$$\epsilon, P \ \text{constant}$$
$$\theta(x,t) = \tau(t)\mathrm{e}^{-r^2}$$
$$\vec{v} = v_x\hat{x}$$
$$\dot{v}_x = 0$$
$$\vec{v}(0) = 0$$

Solution:

$$v_x(x,y,z) = \tanh\left(\frac{c^2}{2}\frac{B^2}{\epsilon+P}\theta\dot{\theta}\mathrm{e}^{r^2}\sqrt{\pi}\mathrm{Erf[x]}\right)$$

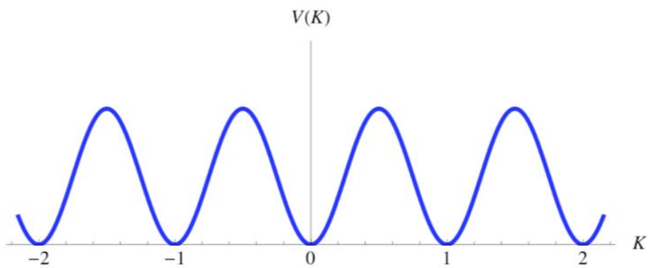




(Kharzeev & Zhitnitsky)

Discussion

- Entropy of the theory with theta-vacuum is conserved. Bloch waves?



- Lowest Landau Levels? For 4 trillion degrees, $k_B T \sim 300MeV$

  $\sqrt{eB} \sim 60MeV$

Quantum Hall Effect condition $\hbar\omega_c \ll k_B T$

Chiral spirals? Charge separation based on Lowest Landau Levels?

Instead, maybe charge separation due to the induced electric field?